Orbiter Technical Notes: Planetary axis precession

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1 Introduction

This document describes the implementation of planetary axis precession in Orbiter.

2 Definitions

The rotation axis of a celestial body is assumed to rotate around a *precession reference axis* at constant obliquity angle and constant angular velocity. Currently, the orientation of the reference axis (OP) is considered time-invariant and is defined with respect to the ecliptic and equinox of J2000 (see Fig. 1). The axis orientation is defined by the obliquity $\varepsilon_{\rm ref}$ (the angle between the axis and the ecliptic north pole, N) and the angle from the vernal equinox Υ to the ascending node of the ecliptic with respect to the body equator, $L_{\rm ref}$. In Orbiter's left-handed system, Υ is defined as (1,0,0), and N is defined as (0,1,0). The rotation matrix $\mathsf{R}_{\rm ref}$ for transforming from ecliptic to precession reference frame is then given by

$$\mathsf{R}_{\rm ref} = \begin{pmatrix} \cos L_{\rm ref} & 0 & -\sin L_{\rm ref} \\ 0 & 1 & 0 \\ \sin L_{\rm ref} & 0 & \cos L_{\rm ref} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varepsilon_{\rm ref} & -\sin\varepsilon_{\rm ref} \\ 0 & \sin\varepsilon_{\rm ref} & \cos\varepsilon_{\rm ref} \end{pmatrix}. \tag{1}$$

The planet's axis of rotation at some time t, OS, is given relative to the reference axis OP, by obliquity $\varepsilon_{\rm rel}$ and longitude $L_{\rm rel}$ (see Fig. 2). $L_{\rm rel}$ is a linear function of time, and is defined as

$$L_{\rm rel}(t) = L_0 + 2\pi\frac{t - t_0}{T_p}, \tag{2}$$

where t_0 is a reference date, L_0 is the longitude at that date, and T_p is the precession period. The rotation from the precession reference frame to the planet's axis frame is described by

$$\mathsf{R}_{\rm rel}(t) = \begin{pmatrix} \cos L_{\rm rel} & 0 & -\sin L_{\rm rel} \\ 0 & 1 & 0 \\ \sin L_{\rm rel} & 0 & \cos L_{\rm rel} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varepsilon_{\rm rel} & -\sin\varepsilon_{\rm rel} \\ 0 & \sin\varepsilon_{\rm rel} & \cos\varepsilon_{\rm rel} \end{pmatrix}. \tag{3}$$

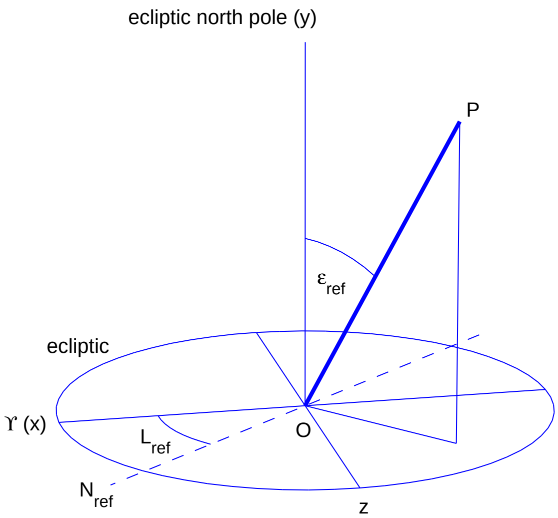

Figure 1: Orientation of the precession reference axis in the ecliptic frame.

The planet's rotation angle $\varphi(t)$ is defined via the siderial period T_s, and a rotation offset φ_0:

$$\varphi(t) = \varphi_0 + 2\pi \frac{t - t_0}{T_s} + [L_0 - L_{\mathrm{rel}}(t)] \cos \varepsilon_{\mathrm{rel}}, \tag{4}$$

where t_0 is a reference time (usually J2000.0). The last term in Eq. 4 accounts for the difference between siderial and node-to-node rotation period. The rotation is encoded in matrix $\mathsf{R}_{\mathrm{rot}}$:

$$\mathsf{R}_{\mathrm{rot}}(t) = \begin{pmatrix} \cos\varphi & 0 & -\sin\varphi \\ 0 & 1 & 0 \\ \sin\varphi & 0 & \cos\varphi \end{pmatrix}. \tag{5}$$

The full planet transformation is the combination of rotation and precession:

$$\mathsf{R}(t) = \mathsf{R}_{\mathrm{ref}} \mathsf{R}_{\mathrm{rel}}(t) \mathsf{R}_{\mathrm{rot}}(t). \tag{6}$$

The direction of the rotation axis is

$$\mathrm{OS} : \mathbf{s}(t) = \mathsf{R}(t) \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}. \tag{7}$$

The resulting axis obliquity and longitude of ascending node are

$$\varepsilon_{\mathrm{ecl}}(t) = \cos^{-1} s_y(t), \qquad L_{\mathrm{ecl}}(t) = \tan^{-1} \frac{-s_x(t)}{s_z(t)}. \tag{8}$$

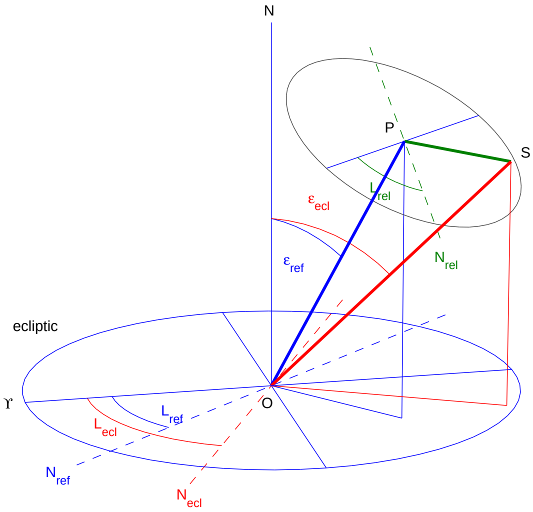

Figure 2: Planet rotation axis.

Figure 3 shows examples of axis obliquity and longitude of ascending node over one precession cycle for different reference obliquities, as a function of L_{rel} (or equivalently, time). Using $\varepsilon_{\mathrm{ecl}}$ and L_{ecl}, an *obliquity matrix* $\mathsf{R}_{\mathrm{ecl}}$ can be defined that rotates from ecliptic to the planet's current precession frame:

$$\mathsf{R}_{\mathrm{ecl}}(t) = \begin{pmatrix} \cos L_{\mathrm{ecl}} & 0 & -\sin L_{\mathrm{ecl}} \\ 0 & 1 & 0 \\ \sin L_{\mathrm{ecl}} & 0 & \cos L_{\mathrm{ecl}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varepsilon_{\mathrm{ecl}} & -\sin\varepsilon_{\mathrm{ecl}} \\ 0 & \sin\varepsilon_{\mathrm{ecl}} & \cos\varepsilon_{\mathrm{ecl}} \end{pmatrix}. \tag{9}$$

Note that like $\mathsf{R}_{\mathrm{ref}}\mathsf{R}_{\mathrm{rel}}$, matrix $\mathsf{R}_{\mathrm{ecl}}$ describes a rotation of the axis from ON to OS. However, there is a difference between the rotation around OS. Specifically, the reference axis for $\mathsf{R}_{\mathrm{ecl}}$ is the ascending node of the ecliptic with respect to the planet equator:

$$\mathrm{ON}_{\mathrm{ecl}} : \mathbf{n}(t) = \mathsf{R}_{\mathrm{ecl}}(t) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \tag{10}$$

The difference between $\mathsf{R}_{\mathrm{ecl}}$ and $\mathsf{R}_{\mathrm{ref}}\mathsf{R}_{\mathrm{rel}}$ can be expressed by an offset matrix $\mathsf{R}_{\mathrm{off}}$:

$$\mathsf{R}_{\mathrm{ecl}}(t)\mathsf{R}_{\mathrm{off}}(t) = \mathsf{R}_{\mathrm{ref}}\mathsf{R}_{\mathrm{rel}}(t), \tag{11}$$

$$\mathsf{R}_{\mathrm{off}}(t) = \mathsf{R}^{T}_{\mathrm{ecl}}(t)\mathsf{R}_{\mathrm{ref}}\mathsf{R}_{\mathrm{rel}}(t). \tag{12}$$

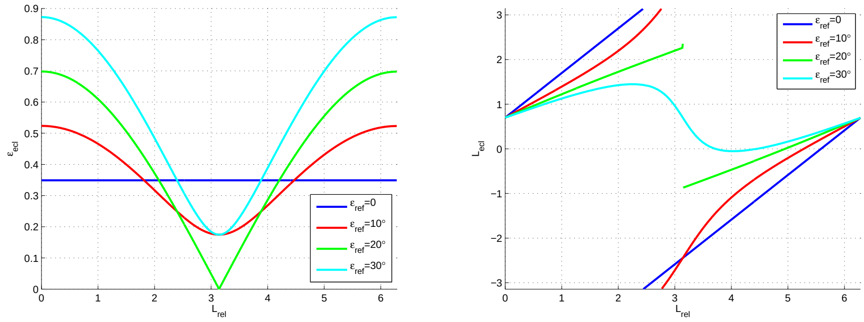

Figure 3: Axis obliquity $\varepsilon_{\rm ecl}$ and longitude of ascending node $L_{\rm ecl}$ as a function of relative longitude $L_{\rm rel}$ over one precession cycle, for four different values of $\varepsilon_{\rm ref}$, and invariant parameters $\varepsilon_{\rm rel} = 20°$, $L_{\rm ref} = 40°$.

$\mathsf{R}_{\rm off}$ describes a rotation around y, so it has the structure

$$\mathsf{R}_{\rm off}(t) = \begin{pmatrix} \cos\varphi_{\rm off} & 0 & -\sin\varphi_{\rm off} \\ 0 & 1 & 0 \\ \sin\varphi_{\rm off} & 0 & \cos\varphi_{\rm off} \end{pmatrix}, \tag{13}$$

and the offset angle $\varphi_{\rm off}$ is given by

$$\varphi_{\rm off}(t) = \tan^{-1}\frac{-[\mathsf{R}_{\rm off}]_{13}}{[\mathsf{R}_{\rm off}]_{11}}. \tag{14}$$

Including this offset into the planet's rotation angle leads to an expression for the planet's rotation angle $r(t)$ with respect to reference direction $\mathbf{n}(t)$:

$$r(t) = \varphi(t) + \varphi_{\rm off}(t). \tag{15}$$

We can now express the full rotation matrix R defined in Eq. 6, using $\mathsf{R}_{\rm ecl}$ and r:

$$\mathsf{R}(t) = \mathsf{R}_{\rm ecl}(t)\tilde{\mathsf{R}}_{\rm rot}(t), \tag{16}$$

where

$$\tilde{\mathsf{R}}_{\rm rot}(t) = \begin{pmatrix} \cos r & 0 & -\sin r \\ 0 & 1 & 0 \\ \sin r & 0 & \cos r \end{pmatrix}. \tag{17}$$

3 Orbiter interface

3.1 Configuration

The precession and rotation parameters supported in planet configuration files are listed in Table 1. The following default assumptions apply:

- If PrecessionObliquity is not specified, $\varepsilon_{\rm ref} = 0$ is assumed. (precession reference is ecliptic normal). In this case, the $L_{\rm ref}$ entry is ignored and $L_{\rm ref} = 0$ is assumed.
- If PrecessionPeriod is not specified, $T_p = \infty$ is assumed (rotation axis is stationary).
- If LAN_MJD is not specified, $t_0 = 51544.5$ is assumed (J2000.0).
- If LAN is not specified, $L_0 = 0$ is assumed.
- If Obliquity is not specified, $\varepsilon_{\rm rel} = 0$ is assumed.
- If SidRotPeriod is not specified, $T_s = \infty$ is assumed (no rotation).
- If SidRotOffset is not specified, $\varphi_0 = 0$ is assumed.

For a retrograde precession of the equinoxes, a negative value of PrecessionPeriod should be used.

parameter	config entry
T_s	SidRotPeriod [seconds]
φ_0	SidRotOffset [rad]
$\varepsilon_{\rm rel}$	Obliquity [rad]
L_0	LAN [rad]
t_0	LAN_MJD [MJD]
T_p	PrecessionPeriod [days]
$\varepsilon_{\rm ref}$	PrecessionObliquity [rad]
$L_{\rm ref}$	PrecessionLAN [rad]

Table 1: Rotation and precession parameter entries in planet configuration files.

3.2 API functions

3.2.1 void oapiGetPlanetObliquityMatrix (OBJHANDLE hPlanet, MATRIX3 *mat)

This function returns $\mathsf{R}_{\rm ecl}(t)$ in Eq. 9 for planet *hPlanet* at the current simulation time.

3.2.2 double oapiGetPlanetObliquity (OBJHANDLE hPlanet)

This function returns $\varepsilon_{\rm ecl}(t)$ in Eq. 8 for planet *hPlanet* at the current simulation time.

3.2.3 double oapiGetPlanetTheta (OBJHANDLE hPlanet)

This function returns $L_{\rm ecl}(t)$ in Eq. 8 for planet *hPlanet* at the current simulation time.

3.2.4 double oapiGetPlanetCurrentRotation (OBJHANDLE hPlanet)

This function returns the current rotation angle $r(t)$ in Eq. 15 for planet *hPlanet* at the current simulation time.

3.2.5 void oapiGetRotationMatrix (OBJHANDLE hPlanet, MATRIX3 *mat)

This function returns $\mathsf{R}(t)$ in Eq. 6 for planet *hPlanet* at the current simulation time.